

Rewrite  $\operatorname{sech}(3\ln 2)$  in terms of exponential functions and simplify.

SCORE: \_\_\_\_\_ / 3 PTS

$$\boxed{\frac{2}{e^{3\ln 2} + e^{-3\ln 2}}} = \frac{2}{e^{\ln 2^3} + e^{\ln 2^{-3}}} = \boxed{\frac{2}{8 + \frac{1}{8}}} \cdot \frac{8}{8} = \boxed{\frac{16}{65}}$$

① POINT EACH

There is an identity involving  $\sinh x$  and  $\cosh x$  that resembles a Pythagorean identity from trigonometry.

SCORE: \_\_\_\_ / 7 PTS

- [a] Write that identity involving  $\sinh x$  and  $\cosh x$ . You do NOT need to prove the identity.

$$\boxed{\cosh^2 x - \sinh^2 x = 1} \quad (1)$$

- [b] Write any two of the three versions of the identity for  $\cosh 2x$ . You do NOT need to prove the identities.

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

or

$$2 \cosh^2 x - 1$$

or

$$2 \sinh^2 x + 1$$

(1) POINT IF YOU GOT  
1 IDENTITY

(2) POINTS IF YOU GOT  
2 (OR 3) IDENTITIES

- [c] If  $\coth x = 3$ , find  $\cosh x$  using identities.

You must explicitly show the use of the identities but you do NOT need to prove the identities.

Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.

$$\left( \frac{1}{2} \right) \boxed{\tanh x = \frac{1}{\coth x} = \frac{1}{3}}$$

$$(1) \boxed{\operatorname{sech}^2 x = 1 - \tanh^2 x} = 1 - \frac{1}{9} = \frac{8}{9} \quad (1/2)$$

$$\left( \frac{1}{2} \right) \boxed{\operatorname{sech} x = \frac{2\sqrt{2}}{3}} \quad (\text{SINCE } \operatorname{sech} x > 0 \text{ FOR ALL } x) \quad (1/2)$$

$$\left( \frac{1}{2} \right) \boxed{\cosh x = \frac{1}{\operatorname{sech} x} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}} \quad (1/2)$$

Prove that  $g(x) = \ln(x + \sqrt{x^2 - 1})$  is the inverse of  $f(x) = \cosh x$  by simplifying  $g(f(x))$ .

SCORE: \_\_\_\_ / 5 PTS

You may use any identities that you found in part [1] of the Hyperbolic Functions Supplement without proving them.

$$\ln(\cosh x + \sqrt{\cosh^2 x - 1}) \quad (1)$$

$$= \ln(\cosh x + \sqrt{\sinh^2 x})$$

$$= \ln(\cosh x + \sinh x) \quad (1\frac{1}{2})$$

$$= \ln e^x \quad (1\frac{1}{2})$$

$$= x \quad (1)$$

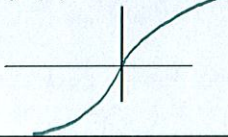


Sketch the general shape and position of the following graphs.  
(Don't worry about specific  $x$  - or  $y$  - coordinates.)

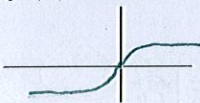
SCORE: \_\_\_\_\_ / 3 PTS

GRADED BY ME

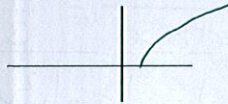
$$f(x) = \sinh^{-1} x$$



$$f(x) = \tanh x$$



$$f(x) = \cosh^{-1} x$$



Write and **prove** a formula for  $\cosh(x-y)$  in terms of  $\sinh x$ ,  $\sinh y$ ,  $\cosh x$  and  $\cosh y$ .

SCORE: \_\_\_\_ / 6 PTS

$$\boxed{\cosh x \cosh y - \sinh x \sinh y}$$

$$= \boxed{\frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}}$$

$$= \boxed{\frac{\cancel{e^{x+y}} + e^{x-y} + e^{-x+y} + \cancel{e^{-x-y}}}{4} - \frac{\cancel{e^{x+y}} - e^{x-y} - e^{-x+y} + \cancel{e^{-x-y}}}{4}}$$

$$= \frac{2e^{x-y} + 2e^{-x+y}}{4}$$

$$= \boxed{\frac{e^{x-y} + e^{-(x-y)}}{2}}$$

① POINT EACH

$$= \cosh(x-y)$$